

Computer Model for Analysis and Re-design of Crystal Filters

Milorad Paskaš, Miroslav Lutovac, Dragi Dujković, Irini Reljin and Branimir Reljin

Abstract – Quartz crystals can be modelled by LC equivalent sections. By using such presentation crystal filters of desired order can be presented as equivalent LC ladder networks. In this paper a method for automated generation of schematic model of crystal filter as a ladder LC structure is described. From schematic model the analysis of filter transfer function can be derived and filter re-design will be possible in order to satisfy desired filter specifications.

Keywords – Crystal filters, LC filter, Quartz filter, Transfer function.

I. INTRODUCTION

DEVELOPMENT of technological techniques for production of microelectronic components with monocrystal quartz units is actual topic. Crystal quartz units used as resonators in crystal oscillators and filters are implemented in various areas in telecommunications such as digital networks, cellular networks and protected/encrypted radio communication networks for military and civilian usage. Since these networks have large spatial coverage and millions of clients it is important to provide high level of synchronization of the system, stability of carry frequencies and appropriate selectivity. All these requirements are achieved with quartz based components.

In this paper we present the first results on computer based analysis of LC lossless filters as the idealized model of crystal filters. Manual calculation of transfer function of higher order LC filters is usually highly demanding and error-prone task. Furthermore computer model of filter enables analysis of impact of parameter variation on transfer function of the filter.

The paper is organized as follows. Chapter two describes equivalent circuit of quartz and crystal filters. Chapters three and four give brief description of Matlab functions for drawing scheme and calculation of transfer

Milorad Paskaš is with the Innovation Center of Faculty of Electrical Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, e-mail: milorad.paskas@gmail.com

Miroslav Lutovac is with Singidunum University, 11000 Belgrade, Serbia, e-mail: lutovac@etf.rs

Dragi Dujković, Irini Reljin and Branimir Reljin are with Faculty of Electrical Engineering, University of Belgrade, Bulevar kralja Aleksandra 73, 11000 Belgrade, Serbia, e-mail: irinitms@gmail.com, {dragi,breljin}@etf.rs

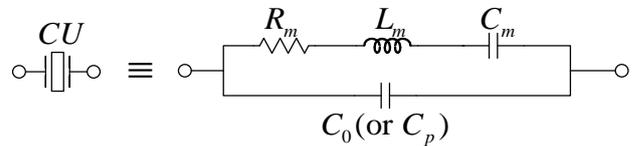


Fig. 1. Equivalent circuit of the crystal unit

function of the LC filter. Applications of these functions are explained as a few examples in chapter five. Finally, chapter six brings conclusions and future work and improvements to be further made.

II. CRYSTAL FILTERS

Crystal unit (CU) is the mechanical vibrating element characterized by *piezoelectricity*: the ability to generate an electric potential in response to applied mechanical distortion, and vice versa. As other mechanical vibrating elements, CU is characterized by its natural frequencies and can be presented by its equivalent electrical scheme as shown in Fig. 1. Series components, R_m , C_m , L_m , are determined by the vibration of the crystal material itself (internal parameters), while the capacitance C_0 models the electrodes on the crystal plate and the stray capacitances from the crystal enclosure (external parameter). Typical plot of reactance, X , vs. frequency, for the crystal unit, is shown in Fig. 2.

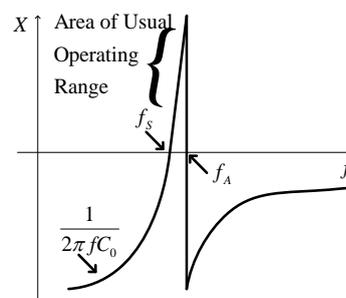


Fig. 2. Reactance X versus frequency for a crystal unit.

The serial (resonance) frequency, f_s , is determined by L_m and C_m , and is given as:

$$f_s = \frac{1}{2\pi\sqrt{L_m \cdot C_m}} \quad (1)$$

while the parallel (antiresonance) frequency, f_A , is:

$$f_A = \frac{1}{2\pi} \sqrt{\frac{C_m + C_0}{L_m C_m C_0}} \cong f_s \left(1 + \frac{C_m}{2C_0}\right) \Big|_{C_0 \gg C_m} \quad (2)$$

Due to extremely high Q-factor, the parallel resonance frequency is only few kilohertz higher than the serial one. Crystals below 30 MHz generally operate between these two frequencies, where high sensitivity, $\Delta X/\Delta f$, is observed. Since in this range inductive nature prevails ($X>0$), any external capacitance will diminish the frequency.

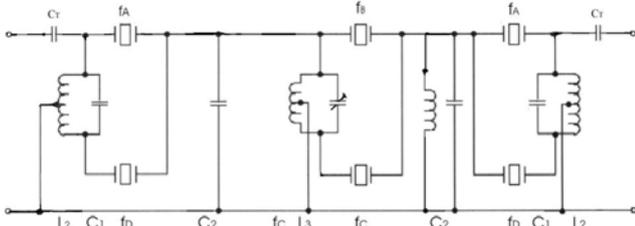


Fig. 3. Electrical scheme of the 50 MHz crystal filter

Manufacturers normally cut and trim crystals to obtain unloaded resonance frequency higher than specified, and by external parallel capacitance the frequency is adjusted to desired value. Crystals above 30 MHz (up to >200 MHz) generally operate at serial resonance, where the impedance is equal to the serial resistance, R_m , which is small (<100 Ω) [1].

Typical crystal filter is shown at Fig. 3. Based on the specifications of the amplitude characteristics in stop and pass band, filter order and corresponding electrical network are determined. The filter is design is based on assumption of real (non ideal) filter elements. Filter design defines the requirements of the crystal specimen. Crystal filters are designed through various nonlinear and graphical methods and sometimes there is no simple method for testing its parameters and amplitude characteristic [2]. By using such equivalent scheme and neglecting resistive loss, crystal filters of desired order are presented with equivalent LC ladder networks.

III. SCHEMATIC REPRESENTATION

The code for schematic representation of lossless LC filters is realised using Schematic Solver [3]. Header and the description of function for LC filter drawing is given:

```
function DrawLC(n,k,x0,y0,dx,s,F)
LC filter drawing.
Inputs:
n - number of nodes;
k - indicator: if k=0 this function
draws Cs and Ls; if k=1 this
function draws Ys and Zs;
x0,y0 - origin coordinates;
dx,s - size of elements in scheme;
F - font size;
Output:
Figure with filter scheme.
```

Therefore filter is drawn only by specifying number of nodes. Value k indicates whether scheme is described by capacitors and inductances or by impedances and admittances. Other input variables consider size of elements, their position in the scheme and font size. An example is given in Figs. 4 and 5 in case of $n=2$.

Function DrawLC is initialized by number of nodes and that information is used to set the position of filter elements (grid). Then nodes, lines, ground, voltage source, resistors, capacitors and inductors are placed within the grid using functions: drawnode, drawlhv, drawnode, drawgrnd, drawvvs, drawres, drawnod, drawcap and drawimp [3], respectively. Input and output sections of the filter are drawn without prior knowledge of number of nodes, while the middle section of the filter is dependent on number of nodes.

LC filter, n=2

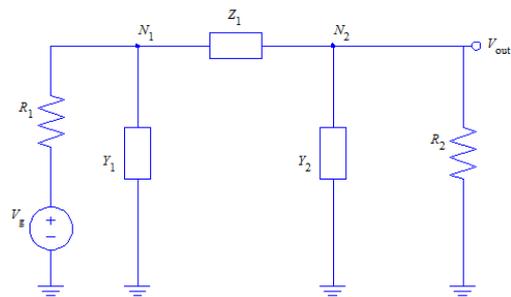


Fig. 4. Representation of LC filter with two nodes: impedance/admittance representation

LC filter, n=2

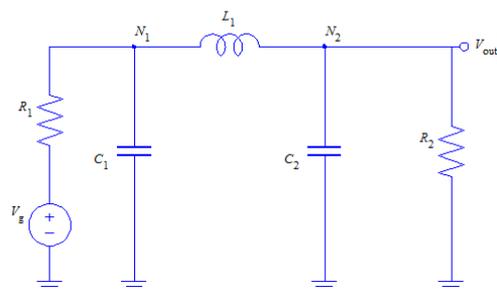


Fig. 5. Representation of LC filter with two nodes: capacitor/inductor representation

IV. TRANSFER FUNCTION CALCULATION

Function TFcalcLC calculates transfer function of the filter and generates its graph. The list of input and output parameters for function calculating transfer function of LC filters is given:

```
Function[H,H1,H2]=TFcalcLC(n,w,R10,R20,L0,C)
Transfer function (TF) calculation in LC
filter.
```

Inputs:
 n - number of nodes;
 w - frequency range(rad/s);
 R10 - input resistor value;
 R20 - output resistor value;
 L0 - 1x(n-1) array with inductors' values;
 C0 - 1xn array with capacitors' values;
 Outputs:
 H - TF calculated w.r.t. R1,R2,Z1..Zn-1,Y1..Yn;
 H1 - TF calculated w.r.t. R1,R2,L1..Ln 1,C1..Cn,s;
 H2 - TF calculated w.r.t. s;
 figure with |H(iw)| plot and H(s) fraction in Latex.

This function is initialized by number of nodes as with the DrawLC function while here the frequency range is needed to be precised. Resistors' values, R10 and R20, all inductors' and capacitors' values, L0 and C0, should be given. Similarly as with the DrawLC function, all filter elements (impedances and admittances) are initialized depending on number of nodes.

After initialization the system of linear equations in Laplace domain is symbolically solved. Solution with impedances/admittances is given as output H and solution with capacitors/inductors is given as output H1. Numerical solution for transfer function is calculated by substitution of numerical inputs in the H1 and substituting 's' with 'jω'. It is plotted and the transfer function with numerical values is given within the figure. An example of using TFcalcLC function is given in Fig. 6.

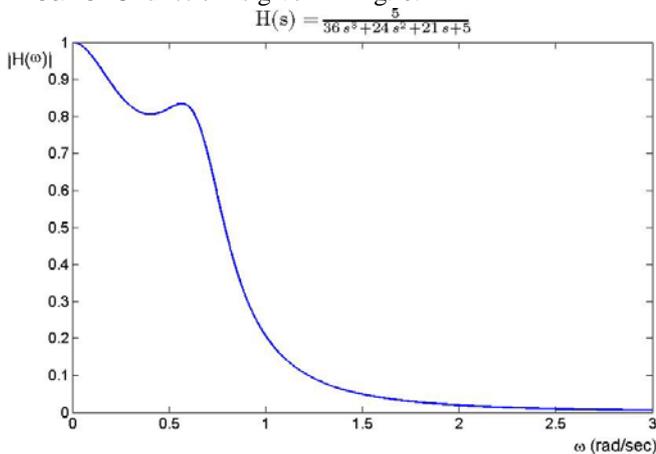


Fig. 6. Transfer function calculated and plotted for filter from Figs. 1 and 2

V. APPLICATIONS

This chapter brings four examples of using functions described in previous two chapters. They illustrate typical use of codes for automatic drawing of LC filters and calculation of transfer function and impact of parameter variation on transfer function.

A. Basics

This example illustrates straightforward application of DrawLC function for visualizing LC filters. Different order LC filters obtained using DrawLC function are shown in Figs. 7, 8 and 9.

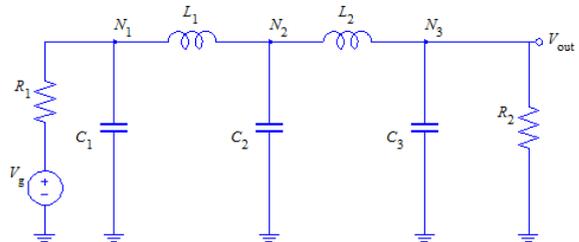


Fig. 7. Representation of LC filter with three nodes (capacitor/inductor representation)

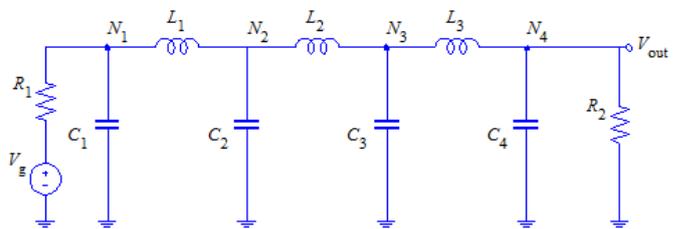


Fig. 8. Representation of LC filter with four nodes (capacitor/inductor representation)

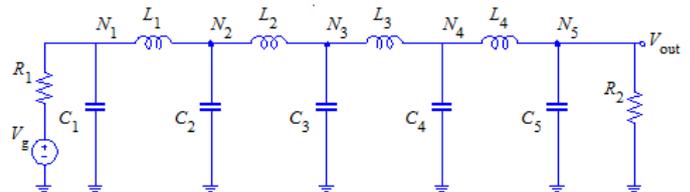


Fig. 9. Representation of LC filter with five nodes (capacitor/inductor representation)

B. Approximation Function

Approximation function [4], [5] is calculated using next lines:

```
M2 = 1/H1/subs(H1,'s',-s)-4;
syms w; M2 = subs(M2,'s',i*w);
pretty(factor(M))
```

As a result there is approximation function (Fig. 10).

$$w^2 (L_1 C_1^2 w^2 - 2 C_1 + L_1)^2$$

Fig. 10. Approximation function of symmetric LC filter with two nodes

Approximation function will be used in example D. Approximation functions for symmetric LC filters with different number of nodes are given in Figs. 11, 12 and 13 in factorized form.

$$w^2 (C_2 C_1^2 L_1^2 w^4 - 2 C_1^2 L_1 w^2 - 2 C_2 C_1 L_1 w^2 + 2 C_1 + C_2 L_1^2 w^2 - 2 L_1 + C_2)^2$$

Fig. 11. Approximation function for symmetric LC filter with three nodes

$$w^2 (L_2 C_1^2 C_2^2 L_1^2 w^6 - 2 C_1^2 C_2 L_1^2 w^4 - 2 L_2 C_1^2 C_2 L_1 w^4 + 2 C_1^2 L_1 w^2 + L_2 C_1^2 w^2 - 2 L_2 C_1 C_2^2 L_1 w^4 + 4 C_1 C_2 L_1 w^2 + 2 L_2 C_1 C_2 w^2 - 2 C_1 + L_2 C_2^2 L_1^2 w^4 + L_2 C_2^2 w^2 - 2 C_2 L_1^2 w^2 - 2 L_2 C_2 L_1 w^2 - 2 C_2 + 2 L_1 + L_2)^2$$

Fig. 12. Approximation function for symmetric LC filter with four nodes

$$w^2 (C_3 C_1^2 C_2^2 L_1^2 L_2^2 w^8 - 2 C_1^2 C_2^2 L_1^2 L_2 w^6 - 2 C_3 C_1^2 C_2 L_1^2 L_2 w^6 + 2 C_1^2 C_2 L_1^2 w^4 - 2 C_3 C_1^2 C_2 L_1 L_2^2 w^6 + 4 C_1^2 C_2 L_1 L_2 w^4 + C_3 C_1^2 L_1^2 w^4 + 2 C_3 C_1^2 L_1 L_2 w^4 - 2 C_1^2 L_1 w^2 + C_3 C_1^2 L_2^2 w^4 - 2 C_1^2 L_2 w^2 - 2 C_3 C_1 C_2^2 L_1 L_2^2 w^6 + 4 C_1 C_2^2 L_1 L_2 w^4 + 4 C_3 C_1 C_2 L_1 L_2 w^4 - 4 C_1 C_2 L_1 w^2 + 2 C_3 C_1 C_2 L_2^2 w^4 - 4 C_1 C_2 L_2 w^2 - 2 C_3 C_1 L_1 w^2 - 2 C_3 C_1 L_2 w^2 + 2 C_1 + C_3 C_2^2 L_1^2 L_2^2 w^6 - 2 C_2^2 L_1^2 L_2 w^4 + C_3 C_2^2 L_2^2 w^4 - 2 C_2^2 L_2 w^2 - 2 C_3 C_2 L_1^2 L_2 w^4 + 2 C_2 L_1^2 w^2 - 2 C_3 C_2 L_1 L_2^2 w^4 + 4 C_2 L_1 L_2 w^2 - 2 C_3 C_2 L_2 w^2 + 2 C_2 + C_3 L_1^2 w^2 + 2 C_3 L_1 L_2 w^2 - 2 L_1 + C_3 L_2^2 w^2 - 2 L_2 + C_3)^2$$

Fig. 13. Approximation function for symmetric LC filter with five nodes

C. Parameter Variation

This example shows the influence of one parameter in LC filter on transfer function. We assume symmetric LC filter with three nodes varying ratio of capacitors values in range 1-5 with step of 0.5. Transfer functions for different values of this parameter are shown in Fig. 14.

This procedure is used in analysis of pass-band width when it is possible to implement quartz crystals with different parameters. Iterative implementation of this analysis should provide crystal with optimal parameters according to amplification (or attenuation) in prescribed frequency range.

D. Inverse Problem

Contrary to previous example herein is illustrated problem of estimating one filter parameter with respect to given criterion. Thus Fig. 15 illustrates optimal inductance values considering amplitude characteristic being one for a range of frequencies around 2 rad/s. The inductance is calculated as the first derivative of approximation function in closed form:

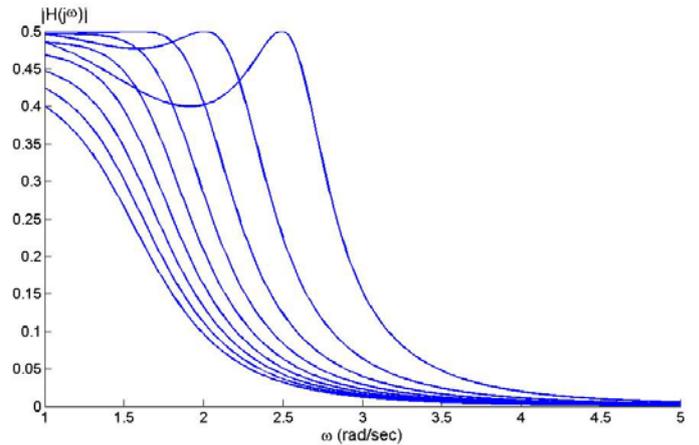


Fig. 14. Transfer functions for different ratios of capacitors values in the symmetric LC filter with three nodes

```
M=subs(H1, 's', li*w)*subs(H1, 's', -li*w);
L = solve(simplify(diff(sqrt(1/M-1))), 'L0');
w = -2.7:0.05:2.7;
L = subs(L, 'w', w);
L = min(L);
```

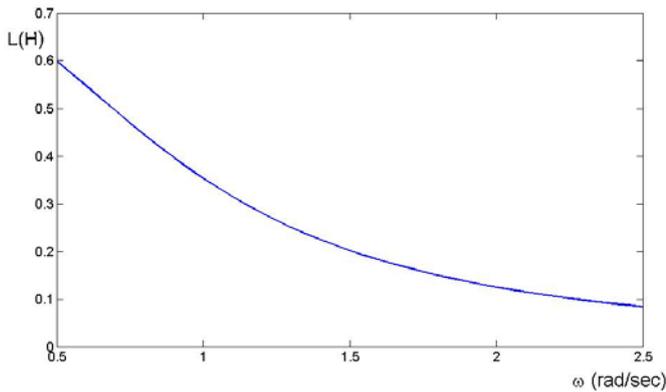


Fig. 15. Optimal inductance values w.r.t. amplitude characteristic being one for a range of frequencies around 2 rad/s

VI. CONCLUSION

In this paper we presented computer codes for drawing and analysing LC lossless filters primarily motivated by crystal filters design. We showed some useful applications of those codes that can be used instead of demanding manual calculations.

This is only the first step in our research towards crystal filter analysis in means of automatization. In our future work we shall consider computer analysis of LC filters with losses. As it was indicated in the paper non ideal model of crystal considers losses introduced by internal resistance of quartz crystal.

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REFERENCES

- [1] Dragi M. Dujković, Dubravka R. Jevtić, Snežana Dedić-Nešić, Lenkica Grubišić, Irini Reljin, Branimir Reljin, "High-quality OCXO for Digital TV" in Proc. *TELSIKS 2009*, vol. 1, pp. 281-284, Nis, Serbia, 2009.
- [2] Dragi M. Dujković, Snežana Dedić-Nešić, Lenkica Grubišić, Branimir Reljin, Irini Reljin, "Crystal Filter 50 MHz for Applications in Specific Environmental Conditions" *TELSIKS 2009*, Vol. 1, pp. 253-256, Nis, Serbia, 2011.
- [3] M. D. Lutovac, D. V. Tošić, *SchematicSolver Version2.2*, 2010: http://books.google.com/books?id=9ue-uVG_JsC
- [4] M. D. Lutovac, V. D. Pavlović, "Symbolic Optimization of Symmetric Lossless Filters", Infoteh 2011, Jahorina, March, 2011.
- [5] V. D. Pavlović, M. D. Lutovac, "Automatizovano projektovanje simetričnih LC lestvičastih filtara za zadati red filtra", *TELFOR 2010*, Beograd, Nov. 23-25, 2010.