Quantitative Analysis of Reactive Power Calculations for Small Non-linear Loads

Marko Dimitrijević and Vančo Litovski

Abstract — In this paper we will present quantitative analysis of reactive power calculated by various definitions. The analysis will be performed on small non-linear loads, such as CFL and LED lamps. All measurements and calculations are realized using virtual instrument for three-phase power factor and distortion analysis.

Keywords — reactive power, virtual instrument

I. Introduction

In linear circuits, with sinusoidal voltages and currents, active, reactive and apparent power are correlated with well-known quadratic formula: $S^2 = P^2 + Q^2$. When nonlinear loads are present one should introduce new quantities in the calculations emanated by the harmonics and related power components [1]. Now, the apparent power includes harmonic components. This is of importance in characterization and design of practical power systems which contain non-linear loads such as switched-mode power supplies [2].

Electronic loads are strongly related to the power quality thanks to the implementation of switched-mode power supplies that in general draw current from the grid in bursts. In that way, while keeping the voltage waveform almost unattached, they impregnate pulses into the current so chopping it into seemingly arbitrary waveform and, consequently, producing harmonic distortions [3]. The current-voltage relationship of these loads, looking from the grid side, is nonlinear, hence nonlinear loads. The existence of harmonics gives rise to interference with other devices being powered from the same source and, having in mind the enormous rise of the number of such loads, the problem becomes serious with serious, sometimes damaging, consequences and has to be dealt with properly.

There are a number of power definitions for non-sinusoidal conditions in order to characterize nonlinear loads and measure the degree of loads' non-linearity. As more general term, non-active power was introduced. All definitions have some advantages over others. Although tend to be general, there is no generally accepted definition.

Marko Dimitrijević and Vančo Litovski are with the Department of Electronics, Faculty of Electronic Engineering, University of Niš, Aleksandra Medvedeva 14, 18000 Niš, Serbia,
E-mail: (marko,vanco)@elfak.ni.ac.rs.

The idea of quantitative analysis of reactive power and power decompositions is presented in the literature [4]. In study [4] widely recognized power decompositions proposed by Budeanu, Fryze, Kimbark, Shepherd and Zakikhani, Sharon, Depenbrock, Kusters and Moore, and Czarnecki are analysed quantitatively. This analysis is performed on simple test circuit.

In this study, we will perform analysis of various reactive power definitions by measuring characteristics of small non-linear loads, such as CFL and LED lamps. First, we will introduce definitions of reactive power proposed by Budeanu, IEEE standard, Kimbark, Sharon, Fryze, Kusters and Moore. Then, the virtual instrument for non-linear loads will be described. Finally, we will present and discuss measured and calculated values.

II. Reactive power definitions

A sinusoidal voltage source

$$v(t) = \sqrt{2} V_{\mathrm{RMS}} \sin(\omega_0 t) \tag{1}$$

supplying a linear load, will produce a sinusoidal current

$$i(t) = \sqrt{2} I_{\mathrm{RMS}} \sin(\omega_0 t - \varphi) \tag{2}$$

where V_{RMS} is the RMS value of the voltage, I_{RMS} is the RMS value of the current, ω is the angular frequency, φ is the phase angle and t is the time. The instantaneous power is

$$p(t) = v(t) \cdot i(t) \tag{3}$$

and can be represented as

$$p(t) = 2 V_{\mathrm{RMS}} I_{\mathrm{RMS}} \sin \omega t \cdot \sin(\omega_0 t - \varphi) = p_p + p_q. \tag{4}$$

Using appropriate transformations we can write:

$$\begin{aligned} p_{\mathrm{p}} &= V_{\mathrm{RMS}} I_{\mathrm{RMS}} \cos\varphi \cdot \left(1 - \cos(2\omega_0 t)\right) = \\ &= P \cdot \left(1 - \cos(2\omega_0 t)\right) \end{aligned} \tag{5}$$

and

$$p_q = -V_{\mathrm{RMS}} I_{\mathrm{RMS}} \sin\varphi \cdot \sin(2\omega_0 t) = -Q \sin(2\omega_0 t) \tag{6}$$

where

$$P = V_{\mathrm{RMS}} I_{\mathrm{RMS}} \cos\varphi,\ Q = V_{\mathrm{RMS}} I_{\mathrm{RMS}} \sin\varphi \tag{7}$$

represent real (P) and reactive (Q) power.

It can be easily shown that the real power presents the average of the instantaneous power over a cycle:

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)\cdot i(t)\cdot dt \tag{8}$$

where t_0 is arbitrary time (constant) after equilibrium, and T is the period (20 ms in European and 1/60 s in American system, respectively).

The reactive power Q is the amplitude of the oscillating instantaneous power p_q. The apparent power is the product of the root mean square value of current times the root mean square value of voltage:

$$S = V_{\mathrm{RMS}} I_{\mathrm{RMS}} \tag{9}$$

or:

$$S = \sqrt{P^2 + Q^2}. \tag{10}$$

In the presence of nonlinear loads the system no longer operates in sinusoidal condition and use of fundamental frequency analysis does not apply any more. The non-sinusoidal voltage and current are expressed by Fourier series:

$$\begin{aligned} v(t) &= V_0 + \sum_{k=1}^{+\infty} \sqrt{2} V_{k,\mathrm{RMS}} \cos\left(k\omega_0 t + \theta_k\right) \\ i(t) &= I_0 + \sum_{k=1}^{+\infty} \sqrt{2} I_{k,\mathrm{RMS}} \cos\left(k\omega_0 t + \psi_k\right). \end{aligned} \tag{11}$$

where $V_{k,\mathrm{RMS}}$ and $I_{k,\mathrm{RMS}}$ represent RMS values, and θ_k and ψ_k phases for k-th harmonic of voltage and current, respectively. V_0 and I_0 represent DC values.

The instantaneous power $p(t)$ calculated by equation (3) can be represented as Fourier series:

$$p(t) = P + \sum_{k=1}^{+\infty} P_k \cos\left(k\omega_0 t + \zeta_k\right) \tag{12}$$

However, expressing components of instantaneous power (P_k, ζ_k) as function of voltage and current spectral components (V_0 , I_0, $V_{k,\mathrm{RMS}}$, $I_{k,\mathrm{RMS}}$, θ_k and ψ_k) in non-sinusoidal conditions is not an easy procedure. The first addend in sum (12), the real power P, determined as constant energy flow and calculated using equations (8) and (11) is

$$\begin{aligned} P &= V_0 I_0 + \sum_{k=1}^{+\infty} I_{k,\mathrm{RMS}} \cdot V_{k,\mathrm{RMS}} \cdot \cos\left(\theta_k - \psi_k\right) \\ P &= P_0 + P_1 + P_{\mathrm{H}} \end{aligned} \tag{13}$$

where P_0, P_1 and P_{H} stands for DC power, active power of fundamental harmonic and harmonic active power, respectively.

There are a number of reactive power definitions and proposed relations with active and apparent power.

A. Budeanu's definition

The most common definition of reactive power is Budeanu's definition, given by following expression for single phase circuit:

$$Q_{\mathrm{b}} = \sum_{k=1}^{+\infty} I_{k,\mathrm{RMS}} \cdot V_{k,\mathrm{RMS}} \cdot \sin\left(\theta_k - \psi_k\right) \tag{14}$$

Budeanu proposed that apparent power is consist of two orthogonal components, active power (13) and non-active power, which is divided into reactive power (14) and distortion power:

$$D_{\mathrm{b}} = \sqrt{S^2 - P^2 - Q_{\mathrm{b}}^2}. \tag{15}$$

B. IEEE Std 1459-2010 proposed definition

IEEE Std 1459-2010 proposes reactive power to be calculated as:

$$Q_{\mathrm{IEEE}} = \sqrt{\sum_{k=1}^{+\infty} I_{k,\mathrm{RMS}}^2 \cdot V_{k,\mathrm{RMS}}^2 \cdot \sin^2\left(\theta_k - \psi_k\right)} \tag{16}$$

Equation (16) eliminates the situation where the value of the total reactive power Q is less than the value of the fundamental component.

C. Kimbark's definition

Similar to Budeanu's definition, Kimbark proposed that apparent power is consist of two orthogonal components, non-active and active power, defined as average power. The non-active power is separated into two components, reactive and distortion power. The first is calculated by equation

$$Q_{\mathrm{k}} = I_{1,\mathrm{RMS}} \cdot V_{1,\mathrm{RMS}} \cdot \sin\left(\theta_1 - \psi_1\right) \tag{17}$$

It depends only of fundamental harmonic. The distortion power is defined as non-active power of higher harmonics:

$$D_{\mathrm{k}} = \sqrt{S^2 - P^2 - Q_{\mathrm{k}}^2}. \tag{18}$$

D. Sharon's definition

This definition introduces two quantities: reactive apparent power, S_{q}, and complementary apparent power S_{c}, defined as:

$$S_{\mathrm{q}} = V_{\mathrm{RMS}} \cdot \sqrt{\sum_{k=1}^{+\infty} I_{k,\mathrm{RMS}}^2 \sin^2(\theta_k - \psi_k)} \tag{19}$$

and

$$S_{\mathrm{c}} = \sqrt{S^2 - P^2 - S_{\mathrm{q}}^2} \tag{20}$$

where S is apparent power (9) and P active power (8).

E. Fryze's definition

Fryze's definition assumes instantaneous current separation into two components named active and reactive currents. Active current is calculated as

$$i_{\mathrm{a}}(t) = \frac{P}{V_{\mathrm{RMS}}^2} v(t) \tag{21}$$

and reactive current as:

$$i_{\mathrm{r}}(t) = i(t) - i_{\mathrm{a}}(t). \tag{22}$$

Active and reactive powers are

$$P = V_{\mathrm{RMS}} \cdot I_{\mathrm{a}}$$
$$Q_{\mathrm{f}} = V_{\mathrm{RMS}} \cdot I_{r} \quad (23)$$

where I_{a} and I_{r} represents RMS values of instantaneous active and reactive currents.

F. Kusters and Moore's power definitions

This definition presents two different reactive power parameters, inductive reactive power

$$Q_{\mathrm{L}} = V_{\mathrm{RMS}} \cdot \frac{\sum_{k=1}^{+\infty} \frac{1}{k} \cdot V_{k,\mathrm{RMS}} \cdot I_{k,\mathrm{RMS}} \cdot \sin(\theta_k - \psi_k)}{\sqrt{\sum_{k=1}^{+\infty} \frac{V_{k,\mathrm{RMS}}^2}{k^2}}} \quad (24)$$

and capacitive reactive power:

$$Q_{\mathrm{C}} = V_{\mathrm{RMS}} \cdot \frac{\sum_{k=1}^{+\infty} k \cdot V_{k,\mathrm{RMS}} \cdot I_{k,\mathrm{RMS}} \cdot \sin(\theta_k - \psi_k)}{\sqrt{\sum_{k=1}^{+\infty} k^2 \cdot V_{k,\mathrm{RMS}}^2}}. \quad (25)$$

III. Virtual instrument for acquisition and reactive power calculations

The measurement and calculation of quantities are performed by measurement setup consists of signal acquisition modules – acquisition subsystem and software support– virtual instrument.

The acquisition and conditioning of the electrical quantities is performed by the acquisition subsystem. It is connected to the power grid from one side, and transfers the power to the load while sampling the values of three voltage and four current signals (Fig. 1). The modules for signal conditioning of the voltage and current waveforms provide attenuation, isolation and anti-aliasing.

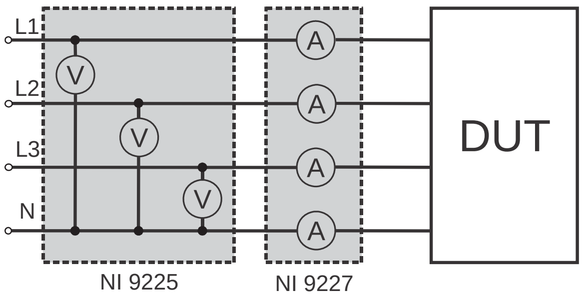

Figure 1. Connection diagram

The acquisition is performed by National Instruments cDAQ-9714 expansion chassis, providing hot-plug module connectivity [5]. The chassis is equipped with two data acquisition modules: NI9225 and NI9227. Extension chassis is connected to PC running virtual instrument via USB interface.

NI9225 has three channels of simultaneously sampled voltage inputs with 24-bit accuracy, 50 kSa/s per channel sampling rate, and 600 V_{RMS} channel-to-earth isolation, suitable for voltage measurements up to 100th harmonic (5 kHz). The 300 V_{RMS} range enables line-to-neutral measurements of 240 V power grids [6].

NI9227 is four channels input module with 24-bit accuracy, 50 kSa/s per channel sampling rate, designed to measure 5 A_{RMS} nominal and up to 14 A peak on each channel with 250 V_{RMS} channel-to-channel isolation [7].

The virtual instrument is realized in in *National Instruments* LabVIEW developing package (Fig. 2), which provides simple creation of virtual instruments. Virtual instruments consist of interface to acquisition module and application with graphic user interface.

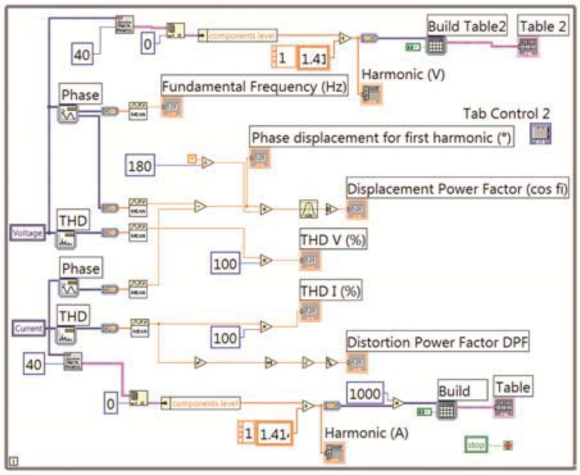

Figure 2. The G code of virtual instrument

Interface to acquisition module is implemented as device driver. cDAQ-9714 expansion chassis is supported by NIDAQmx drivers. All the measurements are performed using virtual channels. A virtual channel is collection of property settings that can include name, a physical channel, input terminal connections, the type of measurement or generation, and scaling information. A physical channel is a terminal or pin at which an analogue signal can be measured or generated. Virtual channels can be configured globally at the operating system level, or using application interface in the program. Every physical channel on a device has a unique name.

For better performance, the main application has been separated into two threads. The first thread has functions for file manipulation and saving measured values. All measured values will be saved in MS Excel file format.

The user interface of the virtual instrument consists of visual indicators. It provides basic functions for measurement. All measured values are placed in a table, and after the measurement process in appropriate file. User interface also provides controls for data manipulation and saving measured values.

IV. Comparison of the calculated values

We have performed measurements on small loads such as various compact fluorescent lamps (CFL, nominal power 7 W – 20 W), indoor light emitting diode lamps

TABLE I

CALCULATED VALUES FOR DIFFERENT REACTIVE POWER DEFINITIONS – CFL LAMPS

No.	Type	Power	P (W)	S (VA)	N (VAR)	Q_b (VAR)	D_b (VAR)	Q_f (VAR)	Q_{IEEE} (VAR)	S_q (VAR)	Q_k (VAR)	Q_C (VAR)	Q_L (VAR)
1	CFL Rod		11.56	17.84	13.58	-6.16	12.10	13.58	6.16	10.24	-6.16	-4.43	-6.11
2	CFL bulb E27	20	17.14	27.72	21.78	-8.43	20.08	21.78	8.43	14.48	-8.43	-6.46	-8.37
3	CFL tube E27	20	16.77	28.46	23.00	-8.44	21.39	23.00	8.45	14.55	-8.45	-6.07	-8.39
4	CFL bulb E27	15	11.59	18.91	14.94	-5.31	13.97	14.94	5.32	9.22	-5.32	-4.00	-5.28
5	Inc E27	100	86.77	86.78	0.80	-0.50	0.63	0.80	0.50	0.56	-0.50	-0.36	-0.49
6	CFL spot E14	7	5.87	9.32	7.25	-2.83	6.67	7.25	2.81	4.23	-2.81	-2.17	-2.80
7	CFL bulb E27	7	6.16	9.86	7.71	-2.64	7.24	7.71	2.65	4.83	-2.65	-2.03	-2.63
8	CFL bulb E14	9	6.46	10.78	8.63	-2.72	8.19	8.63	2.72	5.45	-2.72	-2.08	-2.70
9	CFL tube E14	11	9.89	16.11	12.72	-4.71	11.82	12.72	4.69	7.89	-4.69	-3.61	-4.66
10	CFL tube E27	18	17.10	28.86	23.24	-8.73	21.54	23.24	8.75	13.27	-8.75	-6.64	-8.68
11	CFL tube E27	11	10.63	17.67	14.12	-5.83	12.85	14.12	5.83	8.85	-5.83	-4.41	-5.79
12	CFL helix E27	11	9.58	16.27	13.16	-4.93	12.20	13.16	4.95	8.75	-4.95	-3.68	-4.90
13	Inc E14	60	55.06	55.06	0.61	-0.37	0.49	0.61	0.37	0.37	-0.37	-0.27	-0.37
14	CFL helix E27	18	17.21	28.87	23.18	-8.82	21.43	23.18	8.83	15.55	-8.82	-6.77	-8.76
15	CFL helix E27	20	18.41	30.68	24.54	-9.95	22.43	24.54	9.93	16.14	-9.93	-7.56	-9.86
16	CFL tube E27	15	12.66	21.97	17.95	-6.32	16.80	17.95	6.33	11.63	-6.33	-4.80	-6.28

TABLE II

CALCULATED VALUES FOR DIFFERENT REACTIVE POWER DEFINITIONS – LED LAMPS

No.	Type	Power	P (W)	S (VA)	N (VAR)	Q_b (VAR)	D_b (VAR)	Q_f (VAR)	Q_{IEEE} (VAR)	S_q (VAR)	Q_k (VAR)	Q_C (VAR)	Q_L (VAR)
1	Spot White E27	15	16.92	34.24	29.77	-3.88	29.52	29.77	4.14	20.01	-4.13	-1.98	-4.06
2	Spot White E27	10	13.23	26.33	22.76	-2.97	22.56	22.76	3.17	15.45	-3.17	-1.51	-3.12
3	Bulb W White E27	8	10.00	19.53	16.77	-2.81	16.54	16.77	2.94	11.52	-2.93	-1.74	-2.89
4	Bulb W White E27	6	8.51	9.45	4.11	0.08	4.11	4.11	0.07	3.29	0.07	0.08	0.07
5	Bulb White E27	6	8.69	9.58	4.04	0.09	4.04	4.04	0.08	3.28	0.08	0.08	0.08
6	Bulb White E27	3	4.07	7.70	6.54	-0.84	6.48	6.54	0.90	4.35	-0.90	-0.45	-0.88
7	RGB Change E27	3	1.92	3.17	2.52	0.01	2.52	2.52	0.01	1.39	0.00	0.05	0.00
8	Spot White E14	3	4.00	8.05	6.99	-0.98	6.92	6.99	1.04	4.86	-1.04	-0.52	-1.02

(LED, nominal power 3 W – 15 W), and two incandescent lamp for power reference (60 W and 100 W).

Table 1 shows values for compact fluorescent lamps, as well as two incandescent lamps. Table 2 shows calculated values of reactive power for LED indoor lamps. Following values are displayed: active power (P), apparent power (S), non-active power (N), Budeanu's reactive power (Q_b), Budeanu's distortion power (D_b), Fryze's reactive power (Q_f), IEEE Std 1459-2010 proposed definition for reactive power (Q_{IEEE}), Shanon's apparent power (S_q), Kimbark's reactive power (Q_k), Kusters-Moore's capacitive (Q_C) and inductive (Q_L) reactive power.

Comparison of Budeanu's reactive and distortion power suggests that all examined CFL and LED lamps are non-linear loads ($D_b>Q_b$). Reactive power calculated from Fryze's definition (23) is equal to non-active power, $N=\sqrt{S^2-P^2}$. Kimbark's equation (17) for reactive power, which takes only fundamental harmonic into account, gives approximately ±3% deviance from Budeanu's formula (Q_b). It suggests that the actual contribution of harmonic frequencies to reactive power is small – less than 3% of the total reactive power.

IEEE proposed definition (16) always provides value of the total reactive power greater than the value of the fundamental component.

ACKNOWLEDGEMENT

This research was partly funded by The Ministry of Education and Science of Republic of Serbia within the Project TR32004: “Advanced technologies for measurement, control, and communication on the electric grid”.

REFERENCES

[1] H. W. Beaty, D. G. Fink, *Standard handbook for electrical engineers* (McGraw-Hill, New York, 2007).

[2] John G. Webster, *The Measurement Instrumentation and Sensors Handbook* (CRC Press, 1999).

[3] T. H. Tumiran, M. Dultudes, The Effect Of Harmonic Distortion To Power Factor, *Proceedings of the International Conference on Electrical Engineering and Informatics*, 2007, pp. 834–837 Institute Teknologi Bandung, Indonesia

[4] M. Erhan Balci, M. Hakan Hocaoglu, “Quantitative comparison of power decompositions,” Electric Power Systems Research 78 (2008) 318–329

[5] National instruments cDAQ-9714 Product Data Sheet, National Instruments, http://ni.com

[6] National instruments NI-9225 Product Data Sheet, National Instruments, http://ni.com

[7] National instruments NI-9227 Product Data Sheet, National Instruments, http://ni.com